Sec. 3.1 Intro to Quadratic Functions

Quadratic Function – a function in the form $f(x) = ax^2 + bx + c$ where the domain consists of all real numbers (a not equal to 0) **this is known as standard form

Factored Form: $f(x) = a(x - r)(x - s)$

Graphing Quadratic Functions:

1. Complete the square to get graph in the form above.
2. As $|a|$ get closer to 0, graph gets narrower.
3. As $|a|$ gets larger, graph gets wider.
4. Graphs are parabolas (U shaped figure).
5. Graph opens up if a is positive.
6. Graph opens down if a is negative.
7. Graph will have a maximum if it opens down.
8. Graph will have a minimum if it opens up.
9. Vertex is the lowest or highest point of the graph (also (h, k) when in completed the square form).
10. Axis of symmetry is the equation x = the x value of your vertex

Ex. Graph the function $f(x) = x^2 - x - 6$. Find the vertex and axis of symmetry by factoring.

$f(x) = (x-3)(x+2)$

x-int, when $y=0$ $\Rightarrow$ $0 = (x-3)(x+2)$

$x-3=0 \quad x+2=0$

$x = 3 \quad x = -2$

VERTEX BETWEEN INTERCEPTS: $x = \frac{3+-2}{2} = \frac{1}{2}$

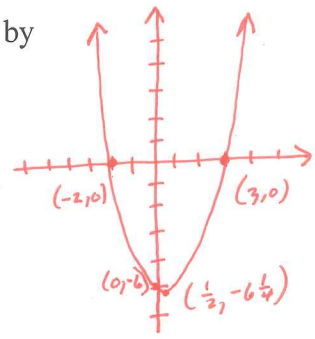
$f(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} - 6$

$= \frac{1}{4} - \frac{2}{4} - 6$

$= -\frac{1}{4} - 6$

$= -6\frac{1}{4}$

$(\frac{1}{2}, -6\frac{1}{4}) \quad x = \frac{1}{2}$



NOTE: If $h = -\frac{b}{2a}$ and $k = \frac{4ac - b^2}{4a}$, then $f(x) = ax^2 + bx + c = a(x - h)^2 + k$.

Vertex – $(-\frac{b}{2a}, f(-\frac{b}{2a}))$

Axis of symmetry – the line $x = -\frac{b}{2a}$

Ex. Find the vertex and axis of symmetry of $f(x) = -3x^2 + 6x + 1$ without graphing. Does the graph open upward or downward?

$a = -3 \quad b = 6$

$x = -\frac{6}{2(-3)}$

$x = -\frac{6}{-6}$

$\boxed{x = 1}$

$f(1) = -3(1)^2 + 6(1) + 1$

$= -3 + 6 + 1$

$= 3 + 1$

$= 4$

$\boxed{(1, 4)}$

opens downward

$f(x) = -3(x^2 - 2x + 1) + 1 + 3$

$f(x) = -3(x-1)^2 + 4$

$(1, 4) \quad x = 1$

Discriminant – $b^2 - 4ac$, the part under the square root sign of the quadratic formula

1. If $b^2 - 4ac > 0$, there will be two distinct x –intercepts and the graph will cross the x-axis in two places.
2. If $b^2 - 4ac = 0$, there will be one x-intercept and the graph will touch the x axis at its vertex.
3. If $b^2 - 4ac < 0$, there are no x-intercepts and the graph does not touch or cross the x-axis.

Ex. Graph $f(x) = 2x^2 - x + 2$ by hand using its vertex, axis, and intercepts.

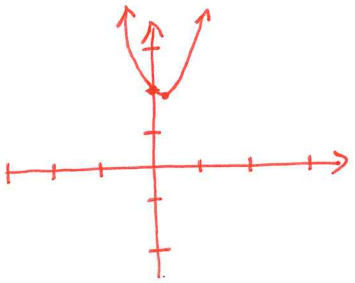
$b^2 - 4ac$
$(-1)^2 - 4(2)(2)$
$1 - 16$
-15
No x-intercepts

$x = \frac{-b}{2a}$
$= \frac{1}{2(2)}$
$x = \frac{1}{4}$

$f(\frac{1}{4}) = 2(\frac{1}{4})^2 - \frac{1}{4} + 2$
$2(\frac{1}{16}) - \frac{1}{4} + 2$
$\frac{1}{8} - \frac{2}{8} + 2$
$-\frac{1}{8} + 2$
$1\frac{7}{8}$

$(\frac{1}{4}, 1\frac{7}{8})$

y-int
$x = 0$
$0 - 0 + 2$
$(0, 2)$



Ex. Find the zeros of $f(x) = x^2 - x - 6$ by using the quadratic formula.

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-6)}}{2}$
$= \frac{1 \pm \sqrt{1 + 24}}{2}$
$= \frac{1 \pm \sqrt{25}}{2}$

$\frac{1+5}{2}$ $\quad$ $\frac{1-5}{2}$
$\frac{6}{2}$ $\quad$ $\frac{-4}{2}$

$x = 3$ or $x = -2$

Ex. A high diver jumps off a 10 meter platform. For t in seconds after the diver leaves the platform until she hits the water, her height h in meters above the water is given by $H = f(t) = -4.9t^2 + 8t + 10$. Find and interpret the domain, range, and intercepts of the graph. Sketch the graph and identify its concavity.

$x = \frac{-b}{2a}$
$= \frac{-8}{2(-4.9)}$
$= \frac{-8}{-9.8}$
$x = .816$

$f(.816) = -4.9(.816)^2 + 8(.816) + 10$
$= 13.27$
$(.82, 13.27)$ – vertex

y-int: $x = 0$
$0 + 0 + 10 = 10$
$(0, 10)$
Height of diving board, at 0 seconds.

x-int
$y = 0$
$= \frac{-8 \pm \sqrt{8^2 - 4(-4.9)(10)}}{2(-4.9)}$
$= \frac{-8 \pm \sqrt{64 + 196}}{-9.8}$
$= \frac{-8 \pm \sqrt{260}}{-9.8}$
$x = -.83$ $\quad$ $x = 2.46$
Outside of domain. $\quad$ Time when diver hits the water.

Domain:
$0 \le t \le 2.46$
Start time of jump and ends when diver hits the water.
Range: $0 \le f(t) \le 13.27$
Water to height of diver at highest height (vertex)

Ex. Given a parabola that has a y-intercept of 6 and x intercepts at 1 and 3, write an equation to represent the graph.

$y = a(x-1)(x-3)$

y-int, $x = 0$ → $6 = a(0-1)(0-3)$
$6 = a(-1)(-3)$
$6 = 3a$
$2 = a$

$y = 2(x-1)(x-3)$
$= 2(x^2 - 4x + 3)$
$y = 2x^2 - 8x + 6$

HW: pg 109-110 #1, 2, 5, 6, 8, 9, 12, 14, 16, 19, 21, 22, 26, 28, 29, 30, 33, 34, 37